out-of-plane case. The resulting position errors due to inaccurate velocity transformation would then be in the range 300–500 ft without normalization or orthogonalization of the computed transformation matrix by the GP computer.

The GP loading requirements to perform the transformation computations every 0.125 sec are very modest. A 400 register memory would be sufficient to store all constants, variables and program words. Even a very conservative computer would have a memory cycle time of about 5 μsec, an add/subtract time of 2 memory cycle times, and a multiply time of perhaps 4 add times. For these instruction speeds the direction cosine integration and velocity transformation requires 4.12 msec of computation. Since this calculation is performed every 125 msec, the computer duty cycle is only 3.3%.

The speed and memory requirements for a GP computer to process the midpoint-g navigation equations every 0.75 sec are trivial. Only about 100 storage registers are required. The midpoint-g computation involves 22 load or store operations, 21 add or subtract operations, and 20 multiplications. Thus, a computer with the speed indicated above would execute this computation in 1.12 msec. But this computation is repeated every 750 msec, so the navigation computation would occupy the GP computer only 1/670 of the time. Virtually all of the computer's capability remains free to do guidance or other computations. The GP computer to perform the navigation (and transformation) computations cannot be sized without considering these additional tasks, since they may well provide the principal computational requirement.

The study shows clearly that a significant computational accuracy penalty is incurred with the partially strapped down system as opposed to the fully stable platform. The computational accuracy penalty must, of course, be properly weighed in the over-all system design considering instrument errors, electronic and mechanical complexity, and mission objectives.

It is also clear that the computer errors propagating during free fall generally dominate those occurring during the terminal re-entry phase. It is, however, necessary to contain the navigation algorithm error during the atmospheric phase by reducing the integration interval from the optimum value determined for the free-fall phase.

DECEMBER 1969 J. SPACECRAFT VOL. 6, NO. 12

Attitude Performance of the GEOS-II Gravity-Gradient Spacecraft

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The GEOS-II satellite employs an extendible boom to provide an earth-pointing equilibrium orientation (by gravity-gradient stabilization) and a magnetically anchored eddy-current damper to dissipate librational energy. Effects of the gravity-gradient forces, orbital eccentricity, thermal distortion of the stabilization boom, solar radiation pressure and geomagnetically induced residual dipole and damper torques are included in the prelaunch design and postlaunch performance analyses (digital simulations of the nonlinear differential equations of motion). The attitude determination system utilizes measurements of the orientation of the sun and the geomagnetic field to obtain an overdetermined orientation solution, so that instrument calibration parameters can be determined. Flight data obtained during two different time periods indicate that the attitude is within 5° of local vertical 96% of the time. General agreement between flight data and the results of the digital simulation is obtained.

Nomenclature

a = semimajor axis
c = damping coefficient
I_x,I_y,I_z = inertia of spacecraft about body-fixed principal axes
I_p,I_r,I_y = inertia of spacecraft about pitch, roll, and yaw axes
l = length of boom
M = mean anomaly
$O_e(x_{ei})$ = Cartesian frame of reference fixed in inertial space and located at the center of mass of the earth (i = 1 to 3)
$O_s(x_{si})$ = Cartesian frame of reference fixed in the spacecraft (i = 1 to 3)
$O_l(x_{li})$ = Cartesian frame of reference used to define the local vertical coordinate system (i = 1 to 3)
α_i = set of Euler angles used to define attitude (i = 1 to 3)
δ_{vert} = total off-vertical libration angle
ϵ = orbit eccentricity
η = angle between the earth-sun line and the normal to the orbit plane
φ = pitch motion
σ = $(I_r - I_y)/I_p$

Received January 6, 1969; revision received October 7, 1969. This work was supported by NASA, Office of Space Science and Applications under Task I of Contract NOw 62-0604-c.

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Introduction

THE repeated success of passive gravity-gradient stabilization (GGS) systems at low altitudes has fostered interest in their use for communication and meterological satellites at higher altitudes. At synchronous altitudes the restoring torque (which is inversely proportional to the cube of the distance of the spacecraft from the center of the earth) is two orders of magnitude smaller than at low altitudes. For this reason, verification of the performance of GGS systems at low altitudes, with the intent to improve the modeling of the

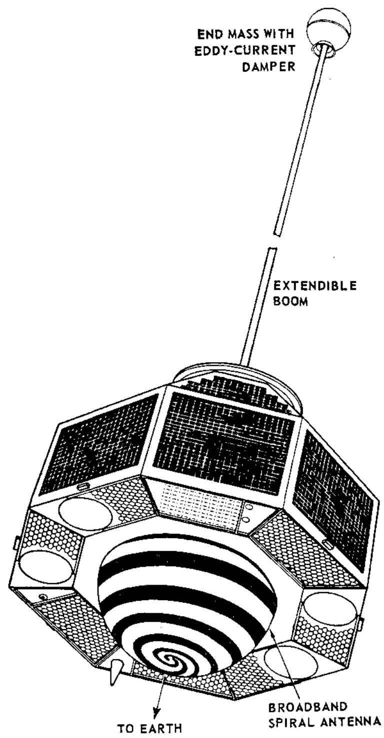

Fig. 1 GEOS-II spacecraft.

control system performance, is important to successful GGS of high-altitude spacecraft.

Passive stabilization and damping offer high reliability and long life. The GEOS-II (Geodetic Earth Orbiting Satellite-II) achieves geocentric stabilization by use of such a system. Onboard measurements determine the orientation of the satellite-sun line and magnetic field vector in the satellite reference system. An earlier paper on GEOS-I compared flight performance with digital simulation results and showed general agreement.[1] However, a failure in part of the attitude detection system somewhat limited that analysis. Hence, flight information from GEOS-II is being used to evaluate the validity of using a digital simulation in performing design analyses.

GEOS-II was designed and built for NASA as part of the National Geodetic Satellite program to study geometric and gravimetric geodesy. The most severe requirement on the stabilization system was imposed by the optical beacon geodetic system, which was designed on the assumption that the deviation from the local vertical δ_{vert} could be kept within 5°. The physical characteristics and geodetic missions of GEOS-II are similar to those of GEOS-I. A schematic of the spacecraft is shown in Fig. 1.

Stabilization System Description

GEOS-II has a single motorized extendible boom manufactured by the DeHavilland Company; at its end is a magnetically anchored eddy-current damper manufactured by the General Electric Company. The flight performance of GEOS-I and prelaunch analyses of GEOS-II indicated that the system would meet the two stabilization requirements: operational status within 15 days after launch and a maximum steady-state libration amplitude of 5°. An important consideration in the selection of this system was that it had been successfully demonstrated by the performance of GEOS-I. The eddy-current damper is a magnet assembly which is free to lock onto the magnetic field of the earth. Rotation of the magnet assembly relative to a conducting spherical shell induces eddy currents in the conductor which impedes the librational motions of the spacecraft. The boom motor is controlled by ground command and backed up by an automatic stop switch activated by detents on the boom tape. This system provides bistable equilibrium about any axis normal to the local vertical. As desired for thermal uniformity, there is no preferred orientation about the longitudinal axis. Proper stabilization can be attained if the extension of the boom is initiated when the boom axis is pointing upwards and within 55° of the local vertical. To insure right-side-up capture before boom extension, an electromagnet whose axis is parallel to the boom axis was added to GEOS-II. Thus, while near the north pole and in view of the command station at the Applied Physics Laboratory, the boom axis would be nearly aligned with the local vertical.

Attitude Determination System

The attitude determination problem is defined mathematically as the evaluation of a transformation matrix which relates the orientation of a reference frame fixed in the spacecraft to some specified reference frame. The nine elements of the matrix are direction cosines and are dependent; however, they can be expressed in terms of three independent Euler angles. For GGS it is natural to choose the reference frame from which the orientation of the spacecraft is measured to be the local vertical reference frame specified by O_t in Fig. 2. The Euler angle set denoted by α_i (i = 1 to 3) is defined by rotations about the successively carried positions of the x_{s3}, x_{s1}, and x_{s2} axes of the O_s reference frame fixed in the spacecraft such that x_{si} is initially parallel to x_{ti} (i = 1 to 3). For small librational amplitudes, the α_i correspond to yaw, roll, and pitch, respectively.

The elements of the transformation matrix can be determined if two vectors (the sun line and the geomagnetic field) are specified simultaneously in both the satellite and local vertical reference systems. These vector components, six scalar quantities in all, provide redundant information for the evaluation of the three independent Euler angles of the transformation matrix.[2] To measure the geomagnetic field components, a three-axis set of second-harmonic flux-gate magnetometers is employed. The satellite-sun line vector components are measured by a system of analog solar cells composed of 11 detectors with approximate cosine response and two linear detectors with designed cutoff at particular solar angles. Four of the cosine detectors are mounted equatorially in mutually orthogonal directions hereafter denoted $+X$, $-X$, $+Y$, $-Y$, and two are mounted in a normally upward direction hereafter denoted Z_1, Z_2. These six detectors provide sufficient information for the determination of attitude as a function of time and for in-orbit calibration of the attitude system.[3] The equatorially mounted detectors were equipped with shades to minimize the effect of the earth's albedo. The five remaining cosine detectors were unshaded and were mounted for determination of the attitude in case the spacecraft was initially captured upside down.

When the satellite is in the earth's shadow, or when the angle between the sun and magnetic field vectors becomes small, the attitude detection system no longer works. However, those two events are easily identified, and data taken during such periods is deleted.

Attitude Simulation

The analytical theory upon which passive GGS is based is well known. The analyses, however, tend to be complicated by the nonlinearity and coupling of the equations of motion and are intractable without simplifying assumptions about the perturbing forces. Furthermore, the nonlinear differential equations governing the large angle motions of a spacecraft must in general be integrated numerically. In the development of the analysis for solution on a digital computer,[4] the spacecraft is considered to have a general configuration with mass an explicit function of time and with mass distribution a function of time and the generalized coordinates and velocities. The center of mass of the satellite is constrained to an eccentric orbit having a longitude of the ascending node and an argument of perigee that are linear functions of time. The attitude is specified in terms of a general set of Euler angles which relate the orientation of a reference frame fixed in the spacecraft to a reference frame whose orientation in inertial space can be specified as an arbitrary function of time.

The resulting Digital Attitude Simulator (DAS) computer program is applicable to a wide class of satellite configurations.[5] It includes the effects of gravity-gradient torques, variable speed rotors, residual magnetic dipoles, solar radiation pressure, thermal bending of gravity-gradient booms, and both passive and semipassive dampers. The eddy-current damper is simulated on the assumption that its magnet assembly is locked to the geomagnetic field. The internal geomagnetic field model used[6] is described in terms of spherical harmonics with a maximum order of seven. For this study, an order of three, which results in a 15-term Legendre polynomial expansion, was used. The DAS thermal bending model for the stabilizing boom includes both bending in the plane of the undeformed boom and the sun and bending normal to the boom-sun plane.[5] Solar radiation pressure forces on the deformed booms, end masses, and spacecraft main body are combined so as to produce the total radiation torque about the instantaneous center of mass of the spacecraft. The effects of aerodynamic torques are negligible at the altitudes of interest. The program also can be used for the analysis of inversion maneuvers by taking into account the time-dependent mass properties of the spacecraft which result from changes in the boom length.

Prelaunch Design Study

A prelaunch design study was made for the nominal orbit which had a perigee of 600 naut miles and an apogee of 800 naut miles. The eccentric orbit, which is undesirable for accurate stabilization, is necessary for accurate satellite geodesy for which the perigee need be well defined. The nominal orbit parameters and mass properties of the spacecraft are given in Table 1. The 15-day interval following launch includes despin from about 1 to 3 rpm to near-orbital rate, erection of the stabilizing boom, and damping to steady

Table 1 Spacecraft and orbit characteristics

Spacecraft characteristics		
Launch configuration		28.1-ft configuration
$I_x = I_y$, slug-ft^2	20.9	$I_x = I_y$ 341.5
I_z, slug-ft^2	25.6	I_z 25.6
Weight, lb	466.0	
Damping coefficient		c = 70,000 dyne-cm-sec
Radius of main body equivalent cylinder		= 63.5 cm
Height of main body equivalent cylinder		= 78.7 cm
Maximum thermal bending tip deflection		= 8.01 cm
Residual magnetic dipole components in spacecraft coordinate system		= 470, 0, 171, pole-cm
Orbit characteristics	Actual	Nominal
Semimajor axis, km	7707.	7674.
Eccentricity	0.032	0.024
Inclination, deg	105.78860	106.0
Ascending node, deg	217.44558	variable
Argument perigee, deg	327.02722	variable
Mean anomaly, deg	0	0
Rate of node, deg/day	1.3983	
Rate of perigee, deg/day	−1.6209	
Time of perigee, day-year, ksec	135–1968 79.839587	variable

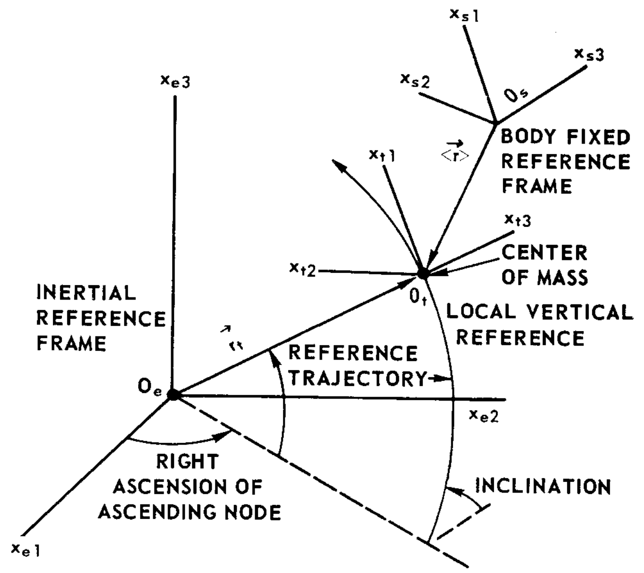

Fig. 2 Reference frames used to derive equations of motion.

state. GEOS-I analyses showed that the time constant of the envelope of the transient response of the eddy-current damper is proportional to I_p/c, as was the despin time constant.[1] The significant difference between GEOS-I and GEOS-II, in terms of expected transient performance, is that I_p/c is smaller for GEOS-II. Thus it was concluded that the performance of GEOS-II would be better than that of GEOS-I, which met the transient performance requirements, and the analyses were not repeated for GEOS-II.

Concurrent with the fabrication of the satellite, a comprehensive DAS analysis of the steady-state performance was initiated. The gravitational field, through the orbit eccentricity, induces a steady-state pitch libration. For small values of eccentricity and small librational amplitudes, the pitch equation is decoupled from the roll and yaw equations. An approximate analytical solution for the pitch motion induced by the orbit eccentricity is[1]

$$\varphi = 2\epsilon \sin M/(3\sigma - 1) \qquad (1)$$

For GEOS-II, and dumbbell satellites in general, $\sigma \approx 1$. Hence, $\varphi \approx \epsilon \sin M$ so that the nominal orbit eccentricity induces a pitch oscillation of 1.38°. The magnetic torques arise from the interaction of the damper and residual magnetization with the geomagnetic field.

Since the damper to be used for GEOS-II had been the backup damper for GEOS-I, its characteristics were fixed to a damping coefficient of 70,000 dyne-cm-sec and a pole strength of 25,000 pole-cm. Thus, boom length and end mass weight were left as the free variables in the steady-state analysis. Studies using the DAS program had shown that the best attitude performance is generally obtained by choosing the end mass weight to be as heavy as is practical and then choosing the boom length based on this end mass weight. Hence, the total weight of the end mass was increased to 11.36 lb. An increase in boom length, which also increases the inertia of the system, tends to minimize the effects of the residual dipole and perturbing effect of the eddy current damper but increases the effects of radiation pressure and thermal bending. Therefore, some optimum boom length exists which minimizes the total steady-state amplitude.

GEOS-I analyses had shown that large roll, or cross-orbit-plane, librations could occur when the orbit was partially shadowed.[1] An examination of the solar radiation pressure forcing function shows that it has a component with twice orbital frequency. Since the natural frequency of roll for this gravity-gradient satellite is also twice orbital, the occurrence of resonance is clear. Further analyses showed the

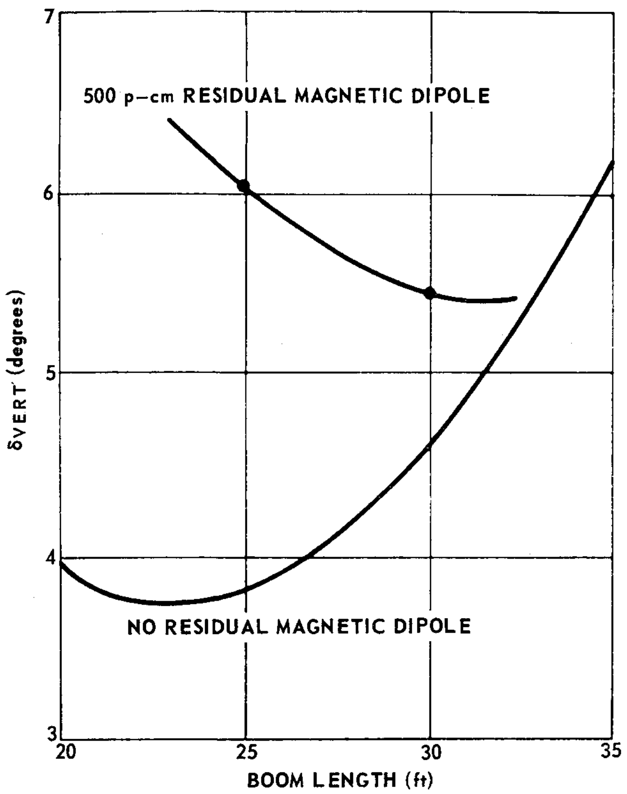

Fig. 3 Prelaunch simulation results for η = 43°.

amplitude of the roll forcing function to be a function of η, the angle between the earth-sun line and the normal to the orbit plane.[1] For circular orbits, the value of η for which the resonance effect is a maximum is given by[7]

$$\eta|_{\substack{\max \\ \text{roll}}} = \cos^{-1}(2a^2 - 1)^{-1/2} \tag{2}$$

where a is the semimajor axis in units of earth radii. For the nominal orbit of GEOS-II, the maximum static roll amplitude induced by solar radiation pressure occurs when $\eta \approx 43.8°$ and when perigee is in the shadow. Since GEOS-II is in a retrograde orbit, the directions of nodal precession and the precession of the sun are the same with respect to a geocentric coordinate system. The magnitudes of the two precession rates are also nearly equal, so that the orientation of the orbit plane with respect to the sun will vary slowly with time, and the condition of roll resonance can persist near its maximum value for several hundred orbits.[7] The DAS results indicated that for perigee in the shadow, a maximum roll amplitude of 4.06° was possible for a 30-ft boom (Table 2). These results

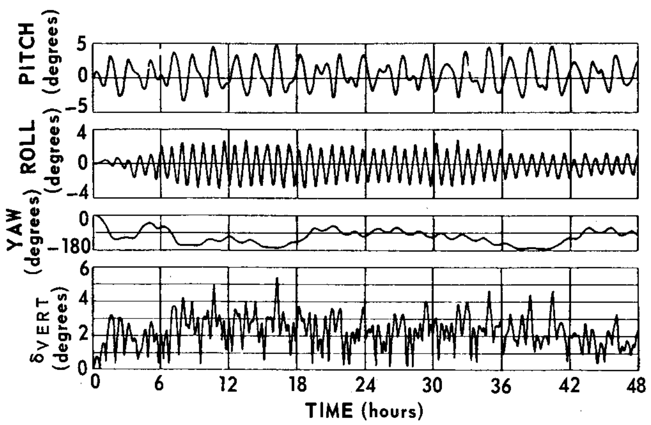

Fig. 4 DAS prelaunch simulation for 30-ft boom; η = 43°, 500 pole-cm dipole.

Table 2 DAS prelaunch results for GEOS-II

Solar angle η, deg	Boom length l, ft	Magnetic dipole, pole-cm	Amplitudes, deg[a]		
			Pitch	Roll	δ_{vert}
43	20	0	3.83	1.37	4.00
	25		3.56	2.48	3.83
	30		3.42	4.06	4.59
	35		3.35	6.05	6.05
	25	500	6.02	2.24	6.03
	30		4.98	3.10	5.43
90	30	500	4.67	1.47	4.75

[a] Maximum values during a period of two days.

(Figs. 3 and 4) indicated that a boom length of near 30 ft would satisfy the steady-state requirements almost 100% of the time under the worst possible case of solar aspect and location of perigee. A detent was then made in the boom tape at 28.1 ft to provide for automatic motor shutoff. Table 1 gives the resulting moments of inertia for the spacecraft with the boom (assumed undeformed) extended.

A preliminary analytical estimate, which neglected gravity-gradient torques, indicated that if the electromagnet failed and the satellite was subsequently captured upside down, the total time required to retract the boom, coast, and extend the boom would permit the operation to be done during a pass over a single station. Starting with the spacecraft initially stabilized upside down, the DAS program was used to simulate an inversion maneuver.

Postlaunch Performance

GEOS-II was launched at 1616 UT on day 11 in 1968 from the Vandenberg Air Force Base. The spacecraft was officially designated 1968 02A and titled by NASA as Explorer XXXVI. The perigee altitude was 583 naut miles and the apogee altitude was 849 naut miles. Despin was successful, and on day 15 at 1224 UT the spacecraft was captured by extending the boom to the 28.1-ft detent.

Near day 20 it appeared that the transient motions had been damped and that the spacecraft was in the steady-state mode. Figures 5–8 show the steady-state attitude performance of the spacecraft during four distinct time periods. In Fig. 5, flight data are shown for days 37–39 (1968), the period shortly after capture. Data for days 68–73 during which time the resonance effect in roll is near its maximum are given in Fig. 6. A more dense set of data was obtained for the periods, days 137 and 255. Attitude for these periods is presented in Figs. 7 and 8. Data received by several ground stations with some degree of mutual visibility were combined to produce a continuous time history of the attitude motion for periods of 40 min; δ_{vert} reaches peak values $\leq 5°$ on both days 137 and 255.

Since the actual ϵ of 0.032 is higher than the nominal prelaunch value of 0.024, pitch oscillations larger than predicted will result. Equation (1) may be used to show that the increase in pitch is ~0.5°. As predicted by the DAS program, the largest librational amplitudes recorded so far occurred when the roll resonance effect was near its maximum.

During the observed life of the satellite, magnetometer measurements, using prelaunch calibrates produced at the facilities of the Naval Ordnance Laboratory, compared

Table 3 Calibration constants for analog solar cosine detectors

Time after launch	$+X$	$-X$	$+Y$	$-Y$	Z_1	Z_2
1 week	0.484	0.484	0.502	0.480	0.559	0.563
4 months	0.428	0.438	0.446	0.430	0.425	0.428

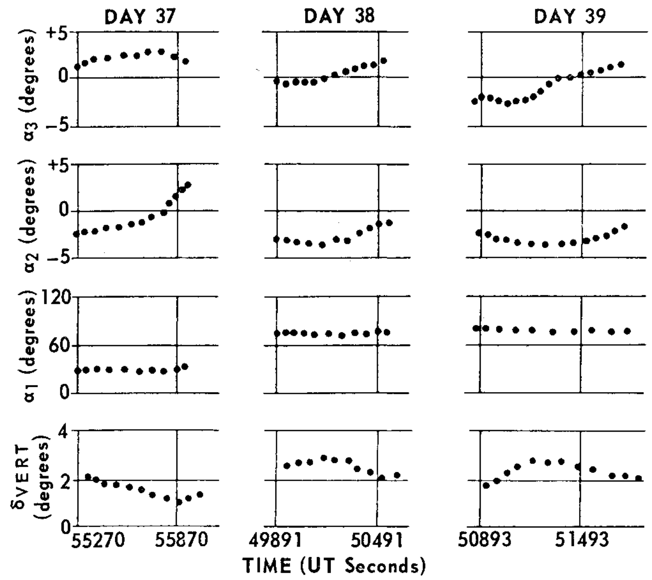

Fig. 5 Flight performance of GEOS-II, three weeks after capture.

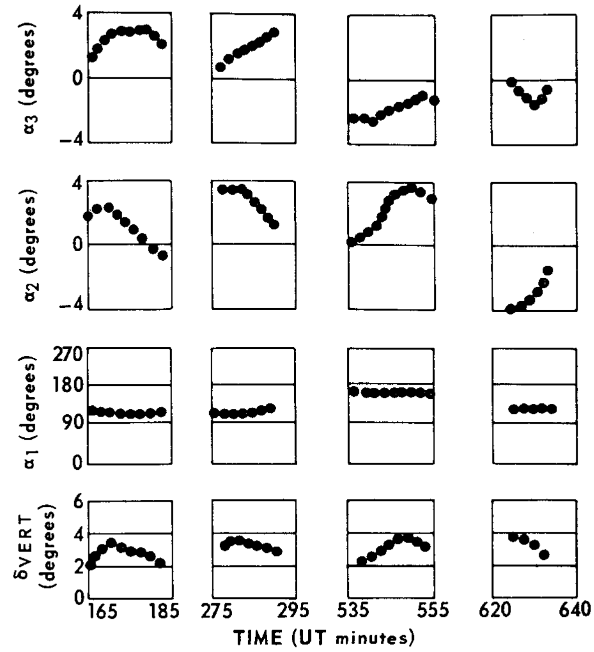

Fig. 7 Flight performance of GEOS-II, day 137 1968.

satisfactorily to within 1% of the expected geomagnetic field. The solar attitude detectors, experiencing an anticipated degradation due to the orbital environment, were recalibrated at intervals to compensate for the associated reduction in calibration constant. Representative results of these in-orbit calibrations are presented as Table 3. The calibration constants listed are the unbiased detector voltages at normal incidence. The dark reference voltages are 0.248 v for the $\pm X$ and $\pm Y$ detectors and 0.249 v for Z_1 and Z_2. In processing the data for days 254 and 255, the calibrations derived from data recorded four months after launch were used and the resultant error in solar vector resolution was less than 1.5%; thus, no significant sensor degradation had occurred. From experience gained during analyses of similar attitude detection systems in the past, no further change in sensor calibrations is anticipated.

Based upon satellite telemetry recorded at APL and from a worldwide network of NASA stations, the attitude of GEOS-II has remained stable to within $\delta_{\text{vert}} = 5°$ almost all of the time. Furthermore, due to the higher ϵ achieved, the upper bound for the expected attitude performance should be increased to 5.5°. The quantitative results given in Table 4 indicate the successful performance of the GEOS-II stabilization system.

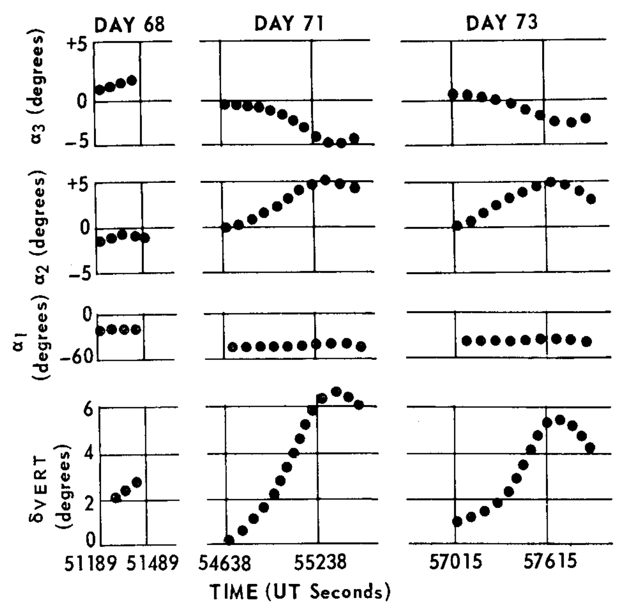

Fig. 6 Flight performance of GEOS-II during a period when the solar induced roll resonance is near its maximum $\eta = 47°$.

Comparison of Theoretical and Experimental Attitude Data

Simulated and experimental attitude motions are compared in Figs. 9 and 10. In each case, the initial conditions required by the numerical simulation were determined only by the experimental data obtained at the beginning of the first segment of data. This is a much more severe test of the accuracy of the simulation than either adjusting the initial conditions to give a best over-all fit, i.e., in a least-squares sense, or treating each segment of data independently. Figure 9 represents a 4-hr interval on day 24 (1968) during which

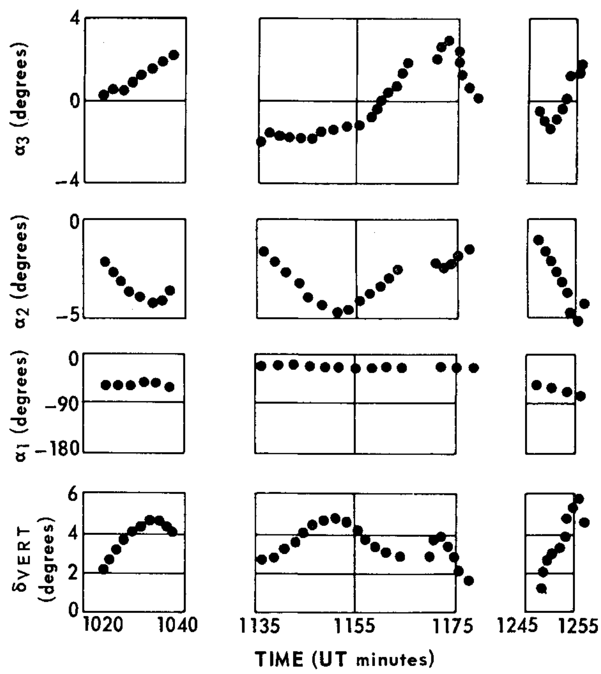

Fig. 8 Flight performance of GEOS-II, day 255 1968.

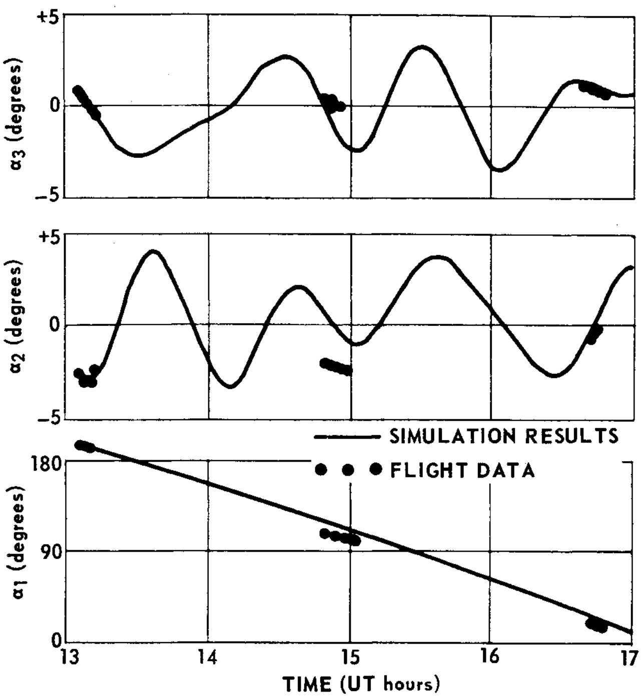

Fig. 9 Comparison of theoretical and experimental data, day 24 1968.

the satellite made two complete orbital revolutions. Rather good agreement is obtained for the first and last segments of data, but there are discrepancies in the α_1 and α_2 motions for the middle segment. Figure 10, for a 6-hr interval on day 136, shows agreement for α_1 and α_3 but discrepancies in α_2. These figures are typical of the results obtained in the postlaunch study, and they indicate that the DAS simulation is adequate to predict the attitude behavior of GEOS-II to better than 0.3° for one pass (i.e., a 15–20 min transit of the satellite over a tracking station). Multirevolution predictions are not as good. The quality of agreement is expected to degrade as the prediction interval increases, because the simulation results represent the integrated effects of errors introduced by approximations in the modeling and in the experimental determination of the initial conditions. Since the over-all accuracy of the attitude determination is no better than 0.5° to 1°, the discrepancies that are observed in

Table 4 Quantitative flight performance of GEOS-II

δ_{vert}	Number of data points[a]	Frequency, %	Cumulative frequency, %
0°–1°	7	1.6	1.6
1°–2°	62	14.3	15.9
2°–3°	130	30.0	45.9
3°–4°	140	32.3	78.2
4°–5°	78	18.0	96.2
5°–6°	14	3.2	99.4
6°–7°	3	0.7	100.0

[a] Data points recorded at one-minute intervals on days 135–138 and 254 and 255, 1968.

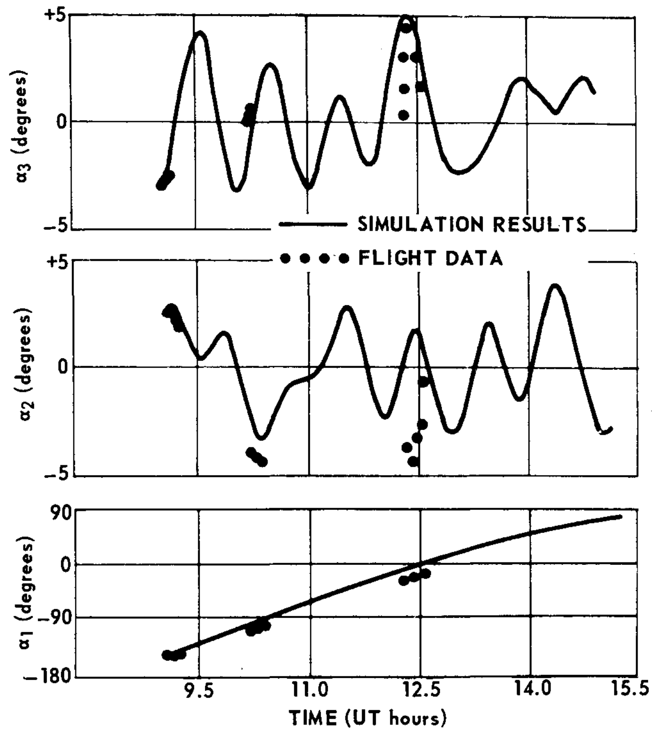

Fig. 10 Comparison of theoretical and experimental data, day 136 1968.

the comparisons are not unexpected. This lower bound of the errors that can be attributed to the attitude determination system precludes the utilization of the experimental results to improve the digital attitude simulation.

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